ASSESSMENT OF GENETIC ALGORITHMS IN CALIBRATION OF AN UNSATURATED SOIL WATER FLOW MODEL APPLIED TO A VERTICAL INFILTRATION EXPERIMENT OF TWO LAYERS OF SOIL

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ABSTRACT: This paper aims at assessing the genetic algorithms for automatic calibration applied to a model that simulates an experimental study on water infiltration in two soil layers. There is the problem of interface between layers of soil. The numerical model of Koide (1990) was used. With the computational progress, the researchers chose to use the inverse model for calibration models. That inverse model was utilized in connection with Genetic Algorithms (GA) that is a mathematical technique that optimizes an Objective Function (OF). The use of a single OF drifts problems of uniqueness. This article uses the collected data from the phenomenon of infiltration of two layers of sandy soils accomplished by Elmaloglou (1980). The results show that the simulation of the experiment, including the interface between layers, is reasonable and there are multiple sets of parameters as responses that calibrate the model.

Key Words: Unsaturated soils, Model of van Genuchten (MVG), Genetic Algorithms (GA)

1. INTRODUCTION

The use of hydrological modeling of water flow in unsaturated soil is an important tool for the management and optimization of water resources (Singh et al., 2010). However, a major obstacle in the application of this model lies in the difficulty of determining their hydraulic properties. Currently, many hydrological models use Richards' equations of water retention in the soil to model the infiltration process. Ines and Droogers (2002) calibrated this type of hydrological model optimizing an objective function (OF). Those authors called for computational power to iterate the equations of the hydrological model using the technique of Genetic Algorithms (GA). Ines and Droogers (2002) drew on data from a mathematical model and evapotranspiration (SWAP). Following this line of research, this study used the mathematical model developed by of Koide (1990), which also solves the Richards' equation using van Genuchten's model (1980) (MVG). The model is tested with data from infiltration experiments as will be discussed in the sequence.

2. INSTRUMENTS ANDMETHODS

2.1 Elmaloglou's Experiment

The performance of the genetic algorithm employed in this paper to calibrate an unsaturated soil water flow model was based upon the results obtained by Elmaloglou (1980) is his classical experiment on water flow in unsaturated soil sample. Elmaloglu (1980) carried out a modeled an experiment of one-dimensional flow of water on two layers of sandy soils. The most important characteristic of this physical phenomenon is the advancement of wetting fronts through the profile of the two layers of sand, especially on the interface of both soils. It is understood that the interface between layers is one of the main sources of error in numerical simulations of this sort. The experiment was performed in a box with a 80cm-depth soil sample. The top layer consisted of 44.5 cm of coarse sand placed over the layer of fine soil. A constant flow rate of 0.78 cm/h was applied on the surface of the sand. The initially dry soil was monitored to determine the changes in volumetric water content along the depth throughout the experiment.

In this experiment, analytical expressions for the water content of soil and hydraulic conductivity were established. The estimation of parameter values for these analytical expressions was obtained by the least-squares method. Elmaloglou (1980) determined the equations for determination of water retention and hydraulic conductivity, Equations 1 and 2, respectively:

$$\theta(\psi) = (\theta_{sat} - \theta_{res}) \frac{\alpha}{\alpha + |\psi|^{\beta}} + \theta_{res} \qquad \psi \leq 0 \qquad [1]$$

$$K(\psi) = K_{sat} \frac{A}{A + |\psi|^{B}} \qquad \psi \leq 0 \qquad [2]$$

where: θ_{sat} denotes volumetric soil water content when soil is saturated ($L^3\ L^{-3}$); θ_{res} is the residual volumetric soil water content ($L^3\ L^{-3}$); ψ is the matric soil head (L); α, β, A and B are dimensionless coefficients; K_{sat} = saturated hydraulic conductivity (L/T). The values of parameters α, β, A and B for the two types of soil obtained by Elmaloglou (1980) are presented in Table 1.

Table 1: Soil parameters (modificated - Elmaloglou, 1980)

Parameter	Coarse sand	Fine sand
θ_{sat} (cm^3/cm^3)	0.270	0.312
θ_{res} (cm^3/cm^3)	0.060	0.208
K_{sat} (cm/h)	18.0	0.72
α	5.641×10^4	6.0359×10^6
β	3.163	3.922
A	3.098×10^4	769.4
B	6.355	2.349

The characteristic curves of the two types of soil based on Equations 1 and parameters of Table 1 are shown in Figure 1. That Figure also shows data points of volumetric soil water content and soil water matric head for both coarse sandy soil and fine sandy soil obtained experimentally by Elmaloglou (1980).

Figure 2 shows the experimental values obtained by Elmaloglou (1980) denoting by points according to the legend. The solid and intermittent lines represent the forward progress of advance of wetting front in the initial time zero and after three, five, seven, nine, and eleven hours simulated by Elmaloglou (1980).

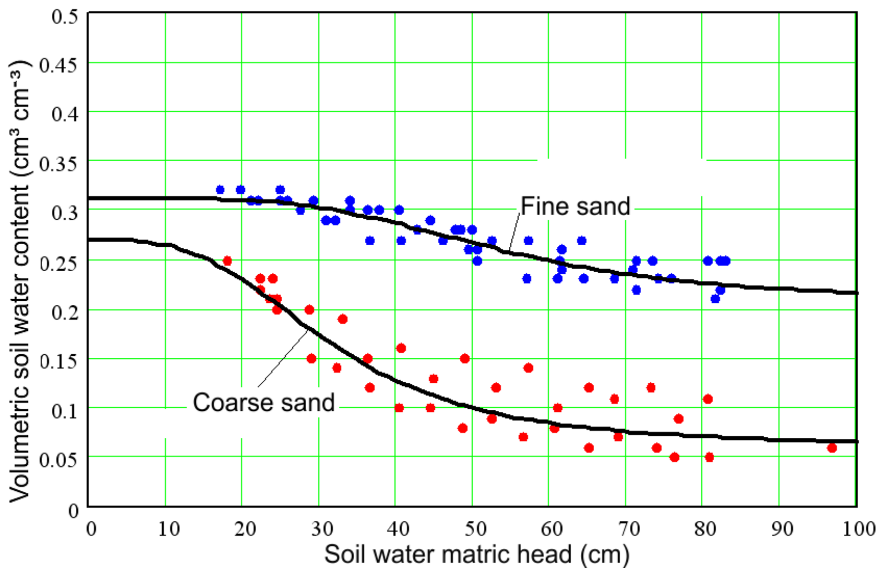

Figure 1: Characteristic curves of coarse soil and fine soil (modificated – Elmaloglou, 1980). The Figure also shows solid points denoting database of volumetric soil water content and soil water matric head obtained experimentally by Elmaloglou (1980)

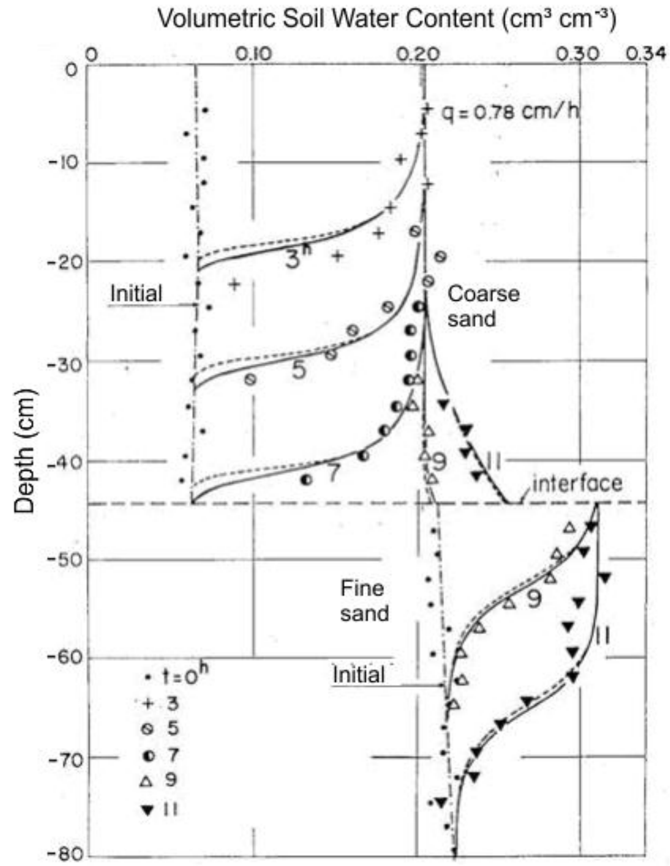

Figure 2: Experimental values with constant flow equal to 0.78 cm/h. Solid and intermittent lines represent the advance of wetting front at different times simulated by Elmaloglou (1980)

2.2 The Model

This current work used van Genuchten (1980) model – MVG (1980) for determination of soil water retention. Those equations are shown in Equations 3, 4a and 4b. The equation of volumetric soil water content in time and in space is shown in Equation 3.

$$\frac{\theta_i^j - \theta_{res}}{\theta_{sat} - \theta_{res}} = \left[\frac{1}{1 + (\alpha|\psi_i^j|)^n}\right]^m \qquad [3]$$

in which: i represents the control point in space; j is the time index; θ_i^j is the volumetric soil water content in j time and in space i; θ_{sat} is the saturated soil; θ_{res} is the residual volumetric soil water content; ψ_i^j is the matric soil head in j time and in space i (L); α e n are dimensionless coefficient; $m = (n-1)/(n)$.

Hydraulic conductivities for the z and x directions, respectively, are computed by:

$$Kz_i^j = Kz_{sat}\left(\frac{\theta_i^j - \theta_{res}}{\theta_{sat} - \theta_{res}}\right)^{1/2}\left\{1 - \left[1 - \left(\frac{\theta_i^j - \theta_{res}}{\theta_{sat} - \theta_{res}}\right)^{1/m}\right]^m\right\}^2 \qquad [4a]$$

$$Kx_i^j = Kx_{sat}\left(\frac{\theta_i^j - \theta_{res}}{\theta_{sat} - \theta_{res}}\right)^{1/2}\left\{1 - \left[1 - \left(\frac{\theta_i^j - \theta_{res}}{\theta_{sat} - \theta_{res}}\right)^{1/m}\right]^m\right\}^2 \quad [4b]$$

wherein: Kz_i^j = hydraulic conductivity in the *z* direction in time *j* in space *i* (L/T); Kx_i^j = hydraulic conductivity in the *x* direction, in time *j*, in space *i* (L/T); Kz_{sat} = saturated hydraulic conductivity in *z* direction (L/T); Kx_{sat} = saturated hydraulic conductivity in the *x* direction (L/T); the other terms have been defined.

Table 2 provides the range of values for each of the six parameters that were used by the search process.

Table 2: Range of values used for the search process

Parameters	n	α	Kx_{sat} (cm/h)	Kz_{sat} (cm/h)	θ_{sat}	θ_{res}
Major value	15.00	0.5000	350.00	350.00	0.50	0.50
Minor value	1.01	0.0001	0.00	0.00	0.00	0.00

The domain of the one-dimensional infiltration experiment was discretized into triangular finite elements as shown in Figure 3. That Figure also shows the fifteen control points which were monitored during the experiment indicating the change in position of the advancing wetting front over time (initial time, three, five, seven, nine, and eleven hours) Boundary conditions of the model are well defined in the experiment.

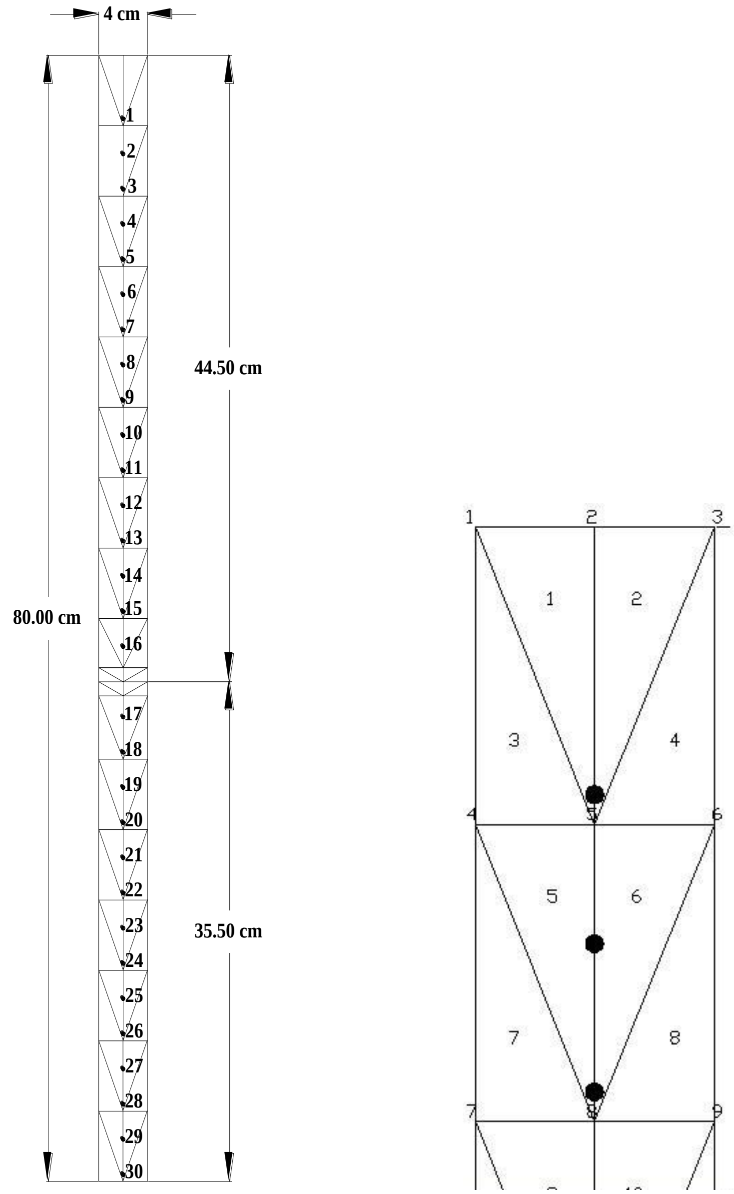

Figure 3: At left shows the mesh adopted for the mathematical model of the Elmaloglou's experiment; at right presents details of the control points

2.3 Calibration Procedure

2.3.1 The Objective Function

The overall goal of the calibration process was obtain a model that was able to simulate properly the soil matric head in the control points over time. Therefore, the Objective Function (OF) employed in the study is the one presented in Equation 5:

$$OF = \frac{1}{1 + \sum_{i=1}^{n} \left| P_{i_{Observed}} - P_{i_{Simulated}} \right|} \qquad [5]$$

where $Pi_{Observed}$ the experimental value of soil matric head established at control point *i* with respect to time and space; $Pi_{Simulated}$ the value of numerically simulated soil matric head at control point *i* with respect to time and space; $\sum_{i=1}^{n}\left|P_{i_{Observed}} - P_{i_{Simulated}}\right|$ resulting from the sum of the absolute differences between the $Pi_{Observed}$ and $Pi_{Simulated}$ at all control points of experiment.

2.3.2 The Genetic Algorithm

Genetic Algorithms (GA) was used to carry out the calibration of the model. The idea of evolutionary computation appeared in the 60, but it was the book of Holland (1975), which gave birth to the Genetic Algorithms (GA) technique. These algorithms use as a model the evolution theory of Charles Darwin: computational elements generating descendants survive to the end of the fittest. Thus, the GA method is a stochastic search of an optimal solution for a cost function. The computational elements called "population" will be evaluated and combined in such a way that the search process should tend to the optimal cost function of finding the descendant of best fitness. The first step of a typical GA is the generation of an "initial population of chromosomes ", which is formed by a random set of chromosomes representing possible solutions to the problem to be solved. During the "evolutionary process", this population is evaluated and each chromosome receives a note, called "fitness", reflecting the quality of the solution that is. In general, the fittest chromosomes are selected and the less fit are discarded. There are three basic operations that occur in traditional GA to create a next generation: (1) selection, (2) crossover and (3) mutation. Each pair of parents creates a child (composed of a set of chromosomes) which results to be a mixture of the parent chromosomes. The mixing process for parents raising children continues to generate a completely new population of population size *N*. Thus the strongest parents will create a generation of children fittest. In practice, the common ability of the population tends to increase with each new generation. The fitness of each child is determined and the process of Selection - Crossover - Mutation (SCM) is repeated. Successive generations are created until the fittest children to be obtained. For Sá (2003), GA are methods with different precepts both conceptually and practically, the GA does not have the same limitations that traditional optimization methods. According to this author, non-convex spaces and discrete variables, for example, do not cause major problems for GA and its performance for multimodal objective functions also tends to be better because the GA sweep the search space in a more comprehensive manner than traditional methods. But in recent years the GA showed difficulties in representing a unique set of responses, as will be demonstrated in this work.

3. RESULTS AND CONCLUSIONS

An initial population of five individuals was used in the GA, in other words, five sets of initial values of input parameters were generated to initialize the search process. Table 3 compares the results of simulated values of the parameters of the equations van Genuchten (1980) with the values of the experimental soil data obtained by Elmaloglou (1980).

Table 3: Comparison of results of numerical simulation parameters with the experimental values obtained by Elmaloglou (1980)

Coarse sand	n	α	Kx_{sat}	Kz_{sat}	θ_{sat}	θ_{res}
Values obtained from the numerical simulation	2.32	0.052	--	9.81	0.29	0.04
Values obtained experimentally	---	---	---	18.0	0.27	0.06

Fine sand	n	α	Kx_{sat}	Kz_{sat}	θ_{sat}	θ_{res}
Values obtained from the numerical simulation	3.05	0.032	--	1.00	0.32	0.20
Values obtained experimentally	---	---	---	0.72	0.31	0.21

Figure 4 compares the retention curves using the parameter values obtained by numerical simulation of Table 3, called "GA Curve Fine Soil" and "GA Curve Coarse Soil", versus the characteristic curves of the model applied by Elmaloglou (1980), called "Elmaloglou Curve Fine Soil" and "Elmaloglou Curve Coarse Soil", respectively. Also that Figure shows the "Upper limit" and "Lower limit" curves that indicates the region of search of Genetic Algorithm that is in connection with Table 2. One can see that the GA curve for coarse soil represents relatively well the retention curve obtained by Elmaloglu (1980) for soil water matric head larger than 30 cm, but deviates from that for values less that 30 cm. For the fine soil, the GA model performed very poorly, with underestimation of volumetric soil water content over the whole range of soil water matric head. Despite these deficiencies in the determination of the retention curves for both soils, the numerical model calibrated with the GA technique provided very good results in representing the profiles of the soil water content over time, as can be seen in Figure 5.

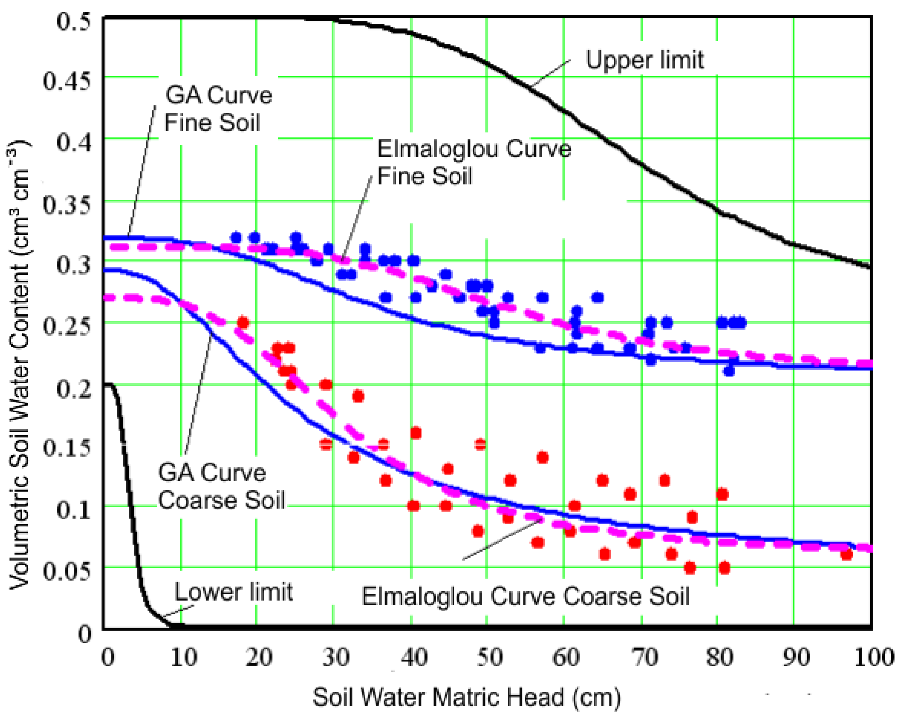

Figure 4: Retention curves obtained by GA technique represented by solid line for both coarse and fine sandy soil. It also shows the curves obtained by Elmaloglou's model represented in dashed lines

Figure 5 shows the location of the control points according to the simulation carried out by Elmaloglou (1980). The solid lines represent the forward progress of advance of wetting front in the initial time zero and after three, five, seven, nine, and eleven hours obtained with GA simulation.

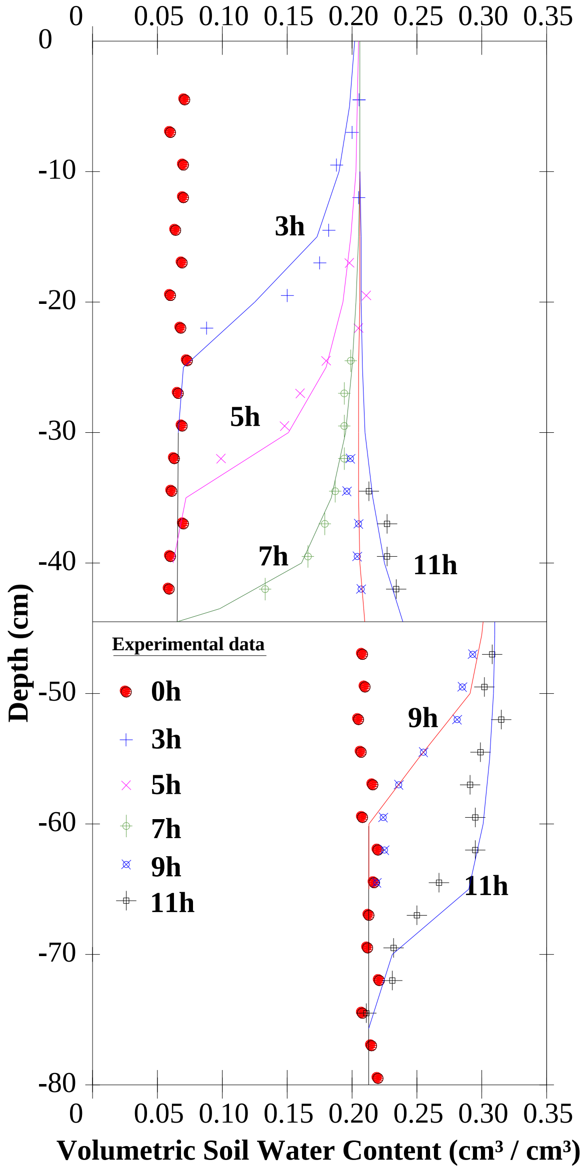

Figure 5: Location of control points. Solid lines represent the simulated profile of volumetric soil water content at different times obtained with the model calibrated with the GA technique.

It is important to point out that many different sets of parameters resulted in very similar performance of the model, a phenomenon well recognized in the literature (Beven, 2005). These similar performances can be seen in Figures 6 and 7. Figure 6 presents three response curves associated with three different sets of parameters, which resulted in very similar values of the OF, while Figure 7 presents the profiles of volumetric soil water content over time. The conclusion is the same here, despite the fact the retention curves are not very well represented, the profiles of the soil water content over time are very well described by the model.

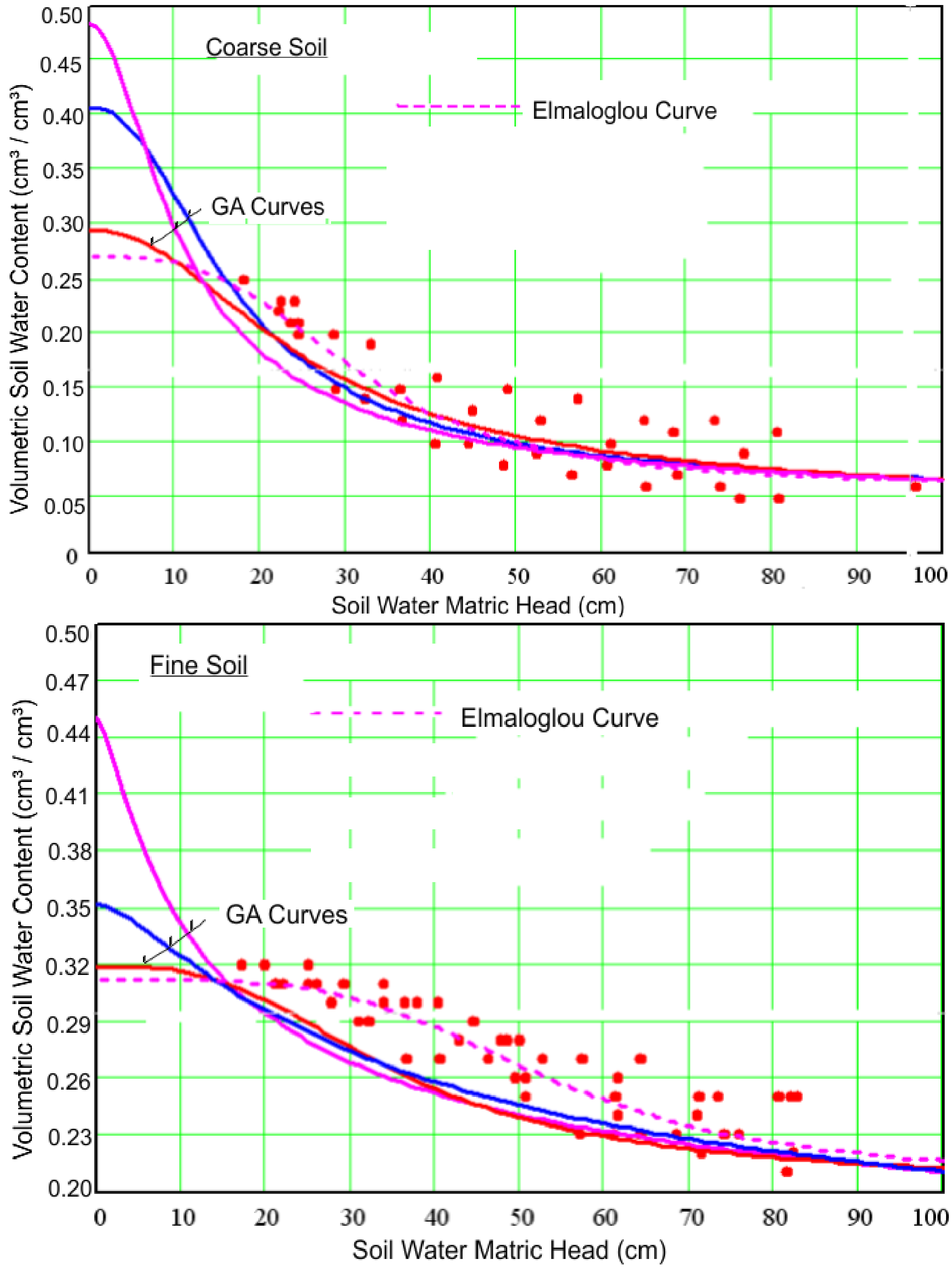

Figure 6: Three retention curves responses in the calibration process for both coarse (upper Figure) and fine soils (lower Figure) obtained by Genetic Algorithms

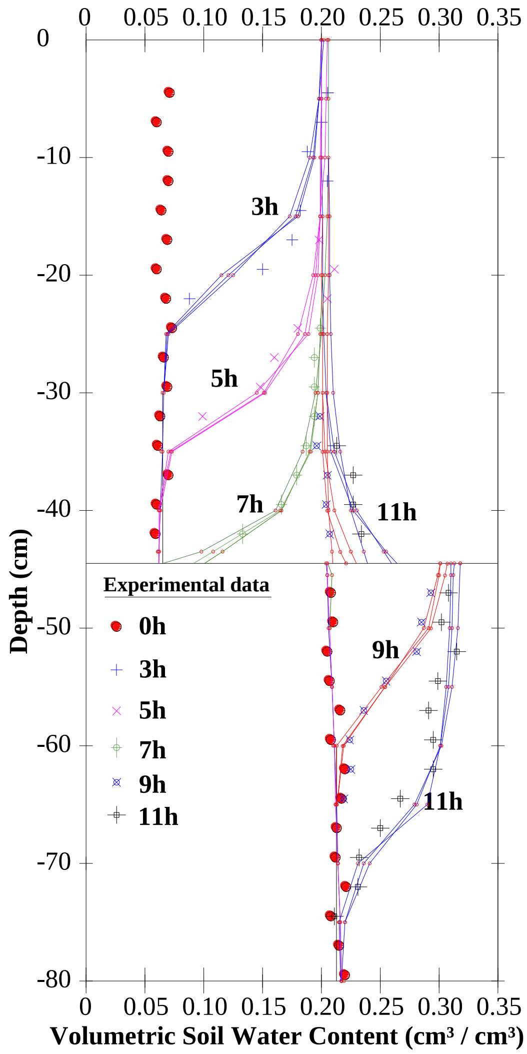

Figure 7: The solid lines represent several curves of advance of wetting front corresponding to different sets of parameters of van Genuchten (1980) obtained with GA simulation. Scale in cm

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