# DISAGGREGATION COEFFICIENTS FOR DAILY MAXIMA BASED ON SIMPLE SCALING INVARIANCE

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**Palavras-chave** — Extreme rainfall, Temporal disaggregation, Scale invariance.

## INTRODUCTION

Long precipitation records are essential for ensuring the reliability of most statistical inference methods. In many engineering applications, estimates of extreme rainfall intensity are required in the form of Intensity–Duration–Frequency (IDF) curves or equations, which are typically derived using Extreme Value Theory (EVT) and probability distributions such as the Generalized Extreme Value (GEV) distribution. However, fitting these distributions using conventional statistical inference methods can be challenging when only short observational records are available, leading to increased uncertainty in the estimation of extreme rainfall (Martins, Stedinger, 2000).

Reliable extreme value analysis depends on accurate representation of the upper tail of the underlying probability distribution. Short records contain relatively few extreme events, which increases uncertainty associated with estimated design quantiles. As a result, daily rainfall records are often used as an alternative data source, once they generally offer longer periods than continuous subdaily observations. Yet, unlike continuous records, which permit the estimation of rainfall depths for arbitrary durations through aggregation in a moving-window, daily rainfall totals must be disaggregated into shorter durations ($d < D$) using empirical scaling relationships or disaggregation ratios.

In Brazil, rainfall disaggregation is still largely based on empirical coefficients proposed by CETESB (1986). Although these coefficients have been widely adopted in engineering practice, their application does not explicitly account for the uncertainty inherent in the disaggregation process. In contrast, studies exploring the scaling properties of rainfall have shown that scaling-based methods provide a physically consistent and promising alternative for estimating subdaily rainfall extremes (Lovejoy, Schertzer, 1986; Menabde *et al.*, 1997; Nguyen, Nguyen, Wang, 1998; Menabde, Seed, Pegram, 1999).

This work explores the scale invariance principle in rainfall intensity and proposes an approach to estimate disaggregation coefficients for precipitation which could be derived at the regional scale. The strategy enables confidence intervals to be estimated, thereby allowing the uncertainty in the disaggregation process to be propagated when estimating subdaily intensities from daily rainfall records and building IDF curves.

## METHODS

We use a set of subdaily gauges located in the state of Rio Grande do Sul, Brazil, from the BR-SDR dataset (Das Neves Almeida *et al.*, 2025). High quality series with at most 20% missing data yearly and at least 10 years were selected and aggregated into hourly durations ($d \in [1, 24]$ by 1 h). Annual maximum series (AMS) were then extracted using a standard calendar year for each $d$.

When dealing with rainfall intensity, the simple scaling hypothesis (Menabde, Seed, Pegram, 1999) assumes intensities at different scales (durations) are equal in distribution except for a scaling factor, defined as the ratio between durations to the power of a scaling exponent $H$ with support $[0, 1]$:

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$$I_d \overset{d}{=} \left(\frac{d}{D}\right)^{-H} I_D. \tag{1}$$

"Simple" in "simple scaling" means this exponent $H$ scales linearly with moment order $q$. In this case, often referred to as "wide sense", $H$ can be estimated by raising both sides to $q$, taking the expected values and finally fitting a linear regression in the log space using $(\log(E[I_d^q]); \log(d/D))$ pairs:

$$E[I_d^q] = \left(\frac{d}{D}\right)^{-qH} E[I_D^q] \quad \Rightarrow \quad \log(E[I_d^q]) = -qH\log(d/D) + \log(E[I_D^q]). \tag{2}$$

Conventional disaggregation coefficients for precipitation depths $C_P$ are estimated from the ratio between precipitation depths at different durations. As $P_d = I_d \times d$, we then have:

$$C_P = \frac{P_d}{P_D} = \frac{I_d}{I_D} \times \frac{d}{D}. \tag{3}$$

Denoting $C_I$ the intensity disaggregation coefficient, a new $C_P$ dependent on $H$ can be derived:

$$C_I = \frac{I_d}{I_D} = \left(\frac{d}{D}\right)^{-H} \quad \Rightarrow \quad C_P = \left(\frac{d}{D}\right)^{-H} \times \frac{d}{D} \quad \Rightarrow \quad C_P = \left(\frac{d}{D}\right)^{1-H}. \tag{4}$$

In order to provide the uncertainties involved in the estimation process, confidence intervals for both $H$ and $C_P$ were generated from 5,000 bootstrap samples. As a preliminary evaluation of the impact of using different estimators of $C_P$ on estimated rainfall intensity quantiles $I_{d,T}$, we define three coefficients: $C_{P,CETESB}$ based on CETESB (1986); $C_{P,R}$ based on the average value of $H$ in Rio Grande do Sul removing a validation site $VAL$ (A801, INMET); and $C_{P,VAL}$ obtained from the validation gauge's own locally estimated $H$. It is worth noting that $C_{P,VAL}$ wouldn't normally be available and is only computed here for the sake of comparison. These different estimators are used to disaggregate a set of quantiles $I_{24h,T}$, estimated by fitting a GEV distribution using L-moments (Hosking, 1990) to $VAL$'s 24 h rainfall record. The associated disaggregated quantiles for $d < 24$ h are plotted together with the observed (empirical) ones at the validation gauge, providing a visual comparison.

## RESULTS

Figure 1a shows the simple scaling model fitted to the validation gauge A801, together with the regional estimate. A good fit and an evident linearity on log space for both models is clear. Figure 1b tells us this behavior is present in the remainder of the dataset, as only one site had an $R^2 < 0.99$.

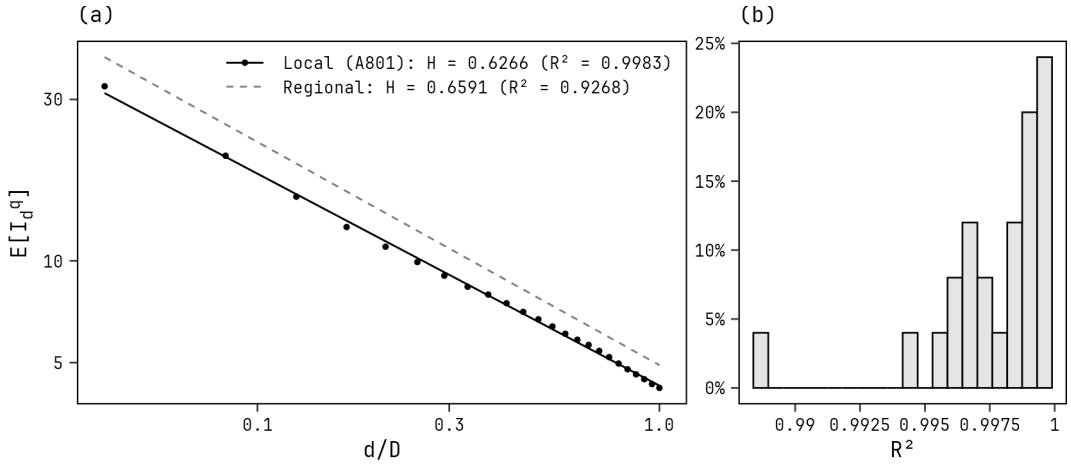



*Figure 1: Simple scaling model for gauge A801 and complete dataset for $q = 1$; and regression $R^2$ values histogram.*

Figure 2 shows resulting $C_P$ estimated for a set of target durations of 1, 6 and 12 h and their confidence intervals. Regional estimates based on estimated group moments of all gauges for each duration and CETESB's coefficients are also shown. The latter has higher values than those estimated from scale invariance. In practice, having higher $C_P$ will lead to higher disaggregated depths, as the temporal structure of the data has a higher slope.

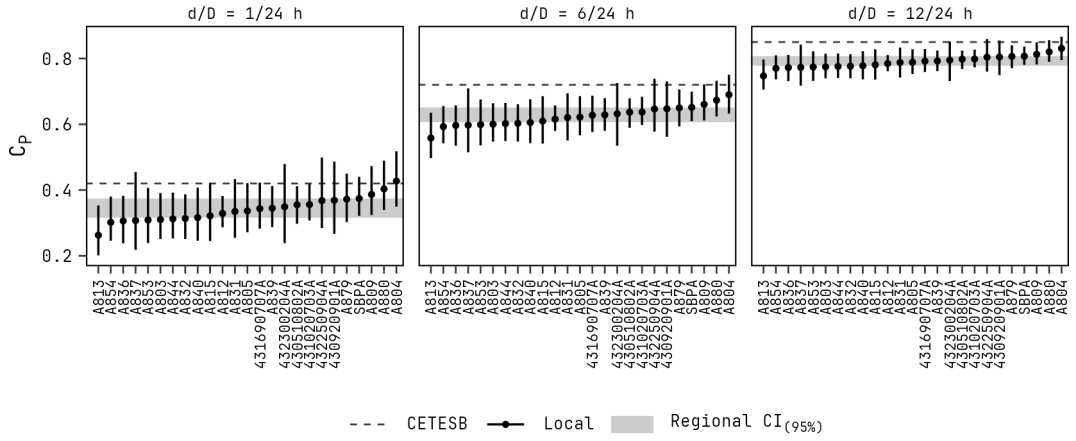



*Figure 2: Simple scaling $C_P$ at site and regionally with bootstrap confidence intervals for $q = 1$, together with CETESB's coefficients.*

Finally, Figure 3 shows IDF curves from a fitted GEV distribution for *VAL* site A801, together with estimated coefficients. Empirical quantiles (from Weibull plotting positions) for the original 1, 6 and 12 h series are shown in solid black points. Dashed lines represent quantiles estimated from disaggregating $I_{24h,T}$ using $C_{P,R}$ (darker grey) and using $C_{P,VAL}$ (lighter gray), both accompanied by respective confidence bands due to their $C_P$ estimation uncertainty. Dotted grey lines represent disaggregated quantiles using $C_{P,CETESB}$. It is noticeable how further from the original IDF CETESB's estimated curves are the smaller the target duration ($d/D$) is, overestimating predicted intensity, while IDFs from simple scaling (*SS*) coefficients, derived regionally and locally, are closer.

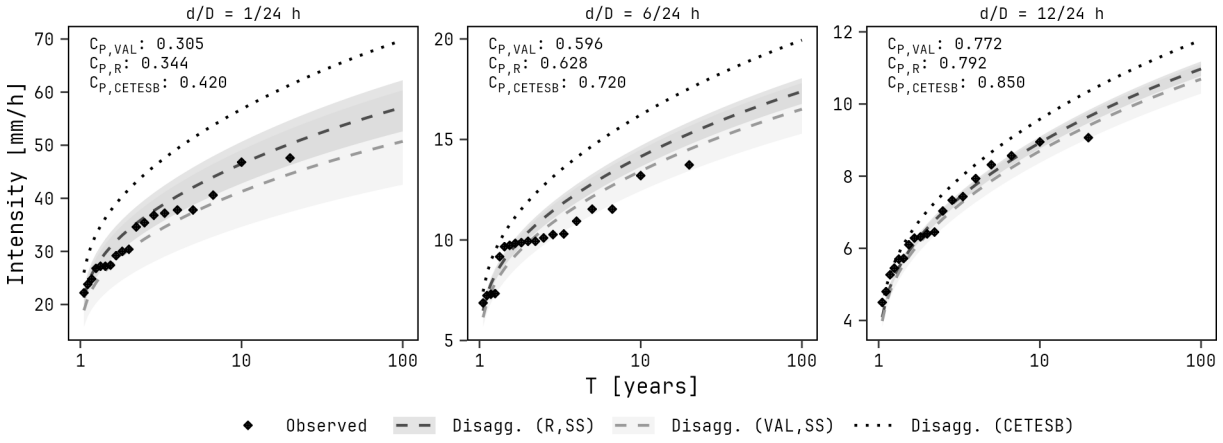



*Figure 3: IDF curves for 1, 6 and 12 h with VAL series and disaggregated series from simple scaling and CETESB coefficients.*

We note that subregional variability is still an important factor that could explain differences between the IDF curves in Figure 3 and the empirical quantiles, and could be masked by grouping all sites into a single region when computing $C_{P,R}$. This is likely seen in Figure 1, where estimated $H_{VAL}$ is smaller than $H_R$, ultimately reducing estimated disaggregated quantiles when employing regional coefficients.

## CONCLUSIONS

This study developed rainfall depth disaggregation coefficients by exploiting the simple scaling properties of annual maximum rainfall series from Rio Grande do Sul, Brazil, for durations in 1–24 h. Results provide strong support for the hypothesis and show that the proposed coefficients better preserve the temporal structure of rainfall than conventional ones currently adopted in engineering practice, which are often generalized across regions despite their inability to adequately represent local rainfall characteristics. Although the validation could be further improved by employing duration-dependent extreme value distributions, the proposed framework demonstrated the potential of scaling-based approaches to derive regionally consistent disaggregation coefficients while quantifying underlying uncertainty. By improving these cofficients in a format familiar to engineering practice, this methodology eases the incorporation of scaling concepts into the estimation of subdaily design rainfall in data-scarce regions.

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