

Estimation of Entrainment Coefficient in Axisymmetric Jets Using Concentration-Based Velocity Derived from LIF Measurements

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Resumo: O arrastamento é um processo fundamental para a compreensão da mistura e da propagação de jatos turbulentos, sendo tradicionalmente avaliado a partir de medições de velocidade. Neste estudo, propõe-se uma abordagem indireta para estimar o coeficiente de arrastamento em um jato turbulento axissimétrico na região dominada pela quantidade de movimento, utilizando dados de concentração obtidos por Laser-Induced Fluorescence (LIF). Devido à ausência de medições diretas de velocidade, a velocidade na linha central foi reconstruída com base na hipótese de proporcionalidade entre os campos de concentração e velocidade, válida principalmente na região autossimilar do jato. A partir da velocidade estimada e da largura característica do jato, o coeficiente de arrastamento foi calculado para três condições experimentais com diferentes números de Reynolds e mesmo diâmetro do bocal. Os resultados indicaram que o coeficiente de arrastamento apresenta flutuações na região próxima ao bocal, mas tende a se estabilizar a jusante, convergindo para valores próximos de 0,055. Esse comportamento está de acordo com a literatura clássica sobre jatos turbulentos axissimétricos e sugere que dados de concentração podem ser utilizados como uma alternativa indireta prática para estimar características de arrastamento quando medições diretas de velocidade não estão disponíveis. No entanto, a abordagem depende da validade da hipótese de proporcionalidade concentração-velocidade e requer validação adicional por técnicas diretas, como PIV ou LDA.

Abstract: Entrainment plays a key role in the mixing and spreading processes of turbulent jets and is commonly characterized using velocity measurements. In this study, the entrainment coefficient in the momentum-dominated region of an axisymmetric turbulent jet is investigated using an indirect approach based on concentration measurements obtained by Laser-Induced Fluorescence (LIF). Due to the absence of direct velocity data, the centerline velocity was estimated from the normalized concentration field by assuming proportionality between concentration and velocity in the self-similar region. The entrainment coefficient was then calculated as a function of the dimensionless distance x/d for three experiments with different Reynolds numbers, while the nozzle diameter was kept constant. The results show that the entrainment coefficients have fluctuations in the near-field region and gradually converges downstream to values close to 0.055, in agreement with classical results for momentum-dominated axisymmetric turbulent jets. These findings suggest that LIF-based concentration measurements can truly provide a practical indirect approach for estimating entrainment behavior when direct velocity measurements are not available. However, the method depends on the validity of the concentration-velocity proportionality assumption and requires further validation using direct velocity measurement techniques such as PIV or LDA.

Palavras-Chave: Arrastamento; Jato axissimétrico; Fluorescência induzida por laser; Estimativa indireta de velocidade; Mistura turbulenta

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INTRODUCTION

Turbulent jets are fundamental free shear flows in fluid mechanics and occur in several engineering and environmental applications, such as pollutant dispersion, wastewater discharge, ventilation, and mixing processes. When a fluid is discharged with an initial momentum into a quiescent ambient, the surrounding fluid is continuously incorporated into the jet. This process, known as entrainment, governs jet spreading, centerline velocity decay, dilution, and volume-flux growth. For this reason, the entrainment coefficient is defined as a key parameter in classical integral models to describe turbulent jet development (Morton et al., 1956; Fischer et al., 1979; Ricou and Spalding, 1961; Pope, 2000).

In the momentum-dominated region of turbulent jets, the flow tends to become self-similar soon after discharge. In this regime, normalized velocity and scalar concentration profiles present similar Gaussian-type distributions, and the entrainment coefficient approaches an approximately constant value. This behavior has been reported in classical experimental and theoretical studies and forms the basis of many analytical and semi-empirical jet models (Wynanski and Fiedler, 1969; Hussein et al., 1994; Jirka, 2004). Previous studies on scalar transport have also shown that concentration fields can reproduce the main decay and spreading trends of turbulent jets, particularly in the self-similar region (Dowling and Dimotakis, 1990; Panchapakesan and Lumley, 1993).

Direct velocity measurements are commonly obtained using techniques such as Particle Image Velocimetry (PIV) or Laser Doppler Anemometry (LDA). Although these methods provide detailed velocity fields, they require specific experimental arrangements and may not always be available. In contrast, Laser-Induced Fluorescence (LIF) provides high-resolution scalar concentration fields with minimal flow disturbance and has been widely applied to turbulent mixing studies (Crimaldi, 2008). Therefore, when direct velocity measurements are unavailable, LIF-based concentration data may provide useful information for estimating velocity-related quantities.

The objective of this study is to evaluate an indirect method for estimating the entrainment coefficient in a momentum-dominated axisymmetric turbulent jet using concentration data obtained from LIF measurements. The centerline velocity is reconstructed from the normalized concentration field based on a proportionality assumption between scalar concentration and velocity, and the entrainment coefficient is then calculated along the dimensionless downstream distance. The method is applied to three experimental cases with different Reynolds numbers and the same nozzle diameter. Since the approach does not include direct validation with PIV or LDA data, the results are interpreted as an indirect estimate of entrainment behavior, with emphasis on the self-similar region where the concentration–velocity similarity assumption is more physically justified.

METHODOLOGY

The entrainment process in turbulent jets refers to the incorporation and mixing of ambient fluid into the jet flow and is one of the primary mechanisms governing jet spreading and mixing. In its most general form, the entrainment rate depends on several parameters, including jet velocity, ambient flow conditions, buoyancy effects, and flow orientation. A generalized expression for the entrainment rate E has been proposed as:

$$E = 2\pi b u_c \left[\alpha_1 + \alpha_2 \frac{\sin \theta}{F_l^2} + \alpha_3 \frac{u_a \cos \theta \cos \sigma}{u_c + u_a} \right] + 2\pi b u_a \sqrt{1 - \cos^2 \theta \cos^2 \sigma} \alpha_4 |\cos \theta \cos \sigma| \quad (1)$$

Where b is the jet width, u_c is the centerline velocity, u_a is the ambient velocity, θ is the jet inclination angle, σ is the ambient flow direction angle, and F_l is the local Froude number. The coefficients α_1 to α_4 represent the contributions of different entrainment mechanisms, including turbulent mixing (α_1), buoyancy effects (α_2), and interactions with ambient flow.

For axisymmetric jets in the momentum-dominated region, the influence of buoyancy and ambient flow can be neglected. Assuming a quiescent ambient ($u_a = 0$), negligible angular effects, and weak buoyancy influence ($Fr \rightarrow \infty$), Eq. (1) reduces to the classical entrainment formulation:

$$E = 2\pi b \alpha U_c \quad (2)$$

Where $\alpha = \alpha_1$ is the entrainment coefficient associated with turbulent pure jet mixing. Using the integral mass conservation for an axisymmetric jet, the entrainment rate is related to the growth of the volume flux Q as:

$$\frac{dQ}{dz} = 2\pi b U_c \quad (3)$$

Since the volume flux scales as $Q \sim b^2 U_c$ the entrainment coefficient (α_{eff}) can be expressed in terms of the stream wise evolution of jet properties as:

$$\alpha_{eff}(x) = \frac{1}{2\pi b(x) U_c(x) dx} \frac{d}{dx} (b(x)^2 U_c(x)) \quad (4)$$

This expression shows that the entrainment coefficient is directly related to the axial variation of the quantity $b^2 U_c$, and therefore requires accurate estimation of both the centerline velocity and the jet width.

Laser-Induced Fluorescence (LIF) Measurements

In this study, concentration data were obtained from previously published experiments conducted by Roberts and Abessi (2014). The experiments employed the Laser-Induced Fluorescence (LIF) technique, a non-intrusive optical method widely used for scalar concentration measurements in turbulent flows. This technique is based on the excitation of fluorescent dye by a laser sheet and the detection of emitted light intensity, which is proportional to the local scalar concentration. The experiments were conducted in a glass tank with dimensions of 6 m (length), 1 m (width), and 0.6 m (depth). A planar LIF system was used to capture instantaneous concentration fields with high spatial resolution (640×480 pixels, corresponding to approximately 1 mm per pixel). The jet flow had fluorescent dye and it illuminated by a laser sheet aligned along the jet axis, while a digital camera recorded the images for further processing. Three experimental cases with different inlet velocities were considered, resulting in different Reynolds numbers, while the nozzle diameter remained constant. The Reynolds number is defined as:

$$Re = \frac{U_0 d}{\nu} \quad (5)$$

Where U_0 is the inlet velocity, d is the nozzle diameter and ν is the kinematic viscosity.

The main experimental parameters, including inlet velocity, Reynolds number, Froude number, cinematic viscosity, Volume Flux, and nozzle diameter, are summarized in Table 1. Since direct velocity measurements were not available, the velocity field was estimated indirectly using the measured concentration data. The initial concentration at the nozzle exit was taken as a reference value $C_0 = 100$, and the centerline concentration decay was analyzed along the downstream direction.

Figure 1 illustrates the geometric behavior of the jet for $Re=1578.6$. The spatial coordinates are presented in dimensionless form as X/d and Y/d , where d is the nozzle diameter.

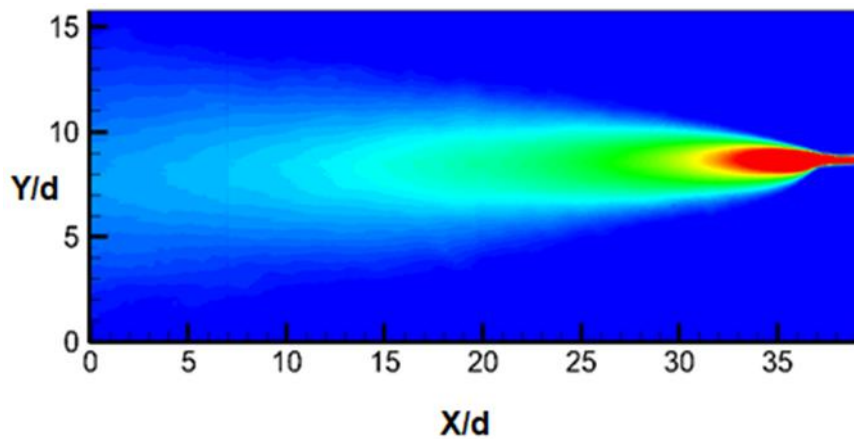


Figure1-Visualization of the turbulent jet concentration field using Laser-Induced Fluorescence (LIF).

To process the data, all concentration values are normalized by the initial concentration, for centerline concentrations c_x :

$$\frac{c_x}{c_0} \quad (6)$$

Assuming similarity between scalar concentration and velocity fields in turbulent jets, a proportional relationship is adopted to estimate the centerline velocity:

$$U_c(x) = U_0 \times \left(\frac{c_x}{c_0}\right) \quad (7)$$

Where U_0 is the initial velocity at the nozzle exit. This simple assumption is consistent with the self-similar behavior of turbulent jets, where both velocity and scalar profiles exhibit similar decay trends along the jet axis. The jet width is also determined based on the linear growth characteristic of axisymmetric turbulent jets in the self-similar region:

$$b(x) = k_b x \quad (8)$$

Where $b(x)$ is the characteristic jet half-width. In turbulent jet theory, the jet width is conventionally defined as the radial distance from the jet centerline at which the local scalar concentration (or velocity) decreases to $1/e$ of its centerline value. This definition is consistent with the Gaussian self-similar profiles observed in fully developed turbulent jets

and is widely adopted in the literature (Jirka, 2004). Under self-similar conditions, the linear growth of $b(x)$ reflects the jet spreading due to turbulent entrainment. Here k_b is the jet spreading rate. In this study, a value of $k_b = 0.1$ is adopted, which is commonly reported for round turbulent jets in the momentum-dominated regime (Jirka, 2004). Using the dimensionless distance, the jet width is expressed as:

$$b(x) = 0.1 \times \left(\frac{x}{d}\right) \times d \quad (9)$$

Based on the estimated centerline velocity and jet width, the variation of flow properties along the jet axis is evaluated. To compute the spatial derivative required in the entrainment formulation, the central difference method is employed. This method provides second-order accuracy and reduces numerical errors when dealing with discrete experimental data, making it more reliable than first-order forward or backward schemes.

$$Q(x) = \frac{d}{dx} (U_c b^2) \Big|_i \approx \frac{(U_c b^2)_{i+1} - (U_c b^2)_{i-1}}{x_{i+1} - x_{i-1}} \quad (10)$$

The above procedure is applied to three experimental cases with different initial velocities and Reynolds numbers, while maintaining a constant nozzle diameter of $d=0.314\text{cm}$. The flow conditions are defined as follows:

Table 1- Summary of experimental conditions and flow parameters

Experiment	Q	d	A	v	U0	Fr	Re
1	$4.10124 \frac{\text{cm}^3}{\text{s}}$	0.314cm	0.077398cm^2	$0.01054 \frac{\text{cm}^2}{\text{s}}$	$52.98906 \frac{\text{cm}}{\text{s}}$	3.019165	1578.612
2	$12.30372 \frac{\text{cm}^3}{\text{s}}$	0.314cm	0.077398cm^2	$0.01054 \frac{\text{cm}^2}{\text{s}}$	$158.9672 \frac{\text{cm}}{\text{s}}$	9.057496	4735.835
3	$20.192 \frac{\text{cm}^3}{\text{s}}$	0.314cm	0.077398cm^2	$0.01054 \frac{\text{cm}^2}{\text{s}}$	$260.8858 \frac{\text{cm}}{\text{s}}$	14.86453	7772.119

Accordingly, the entrainment coefficient is calculated as a function of the dimensionless distance x/d for each case and is used as a basis for evaluating the validity of the concentration-based approach.

RESULT AND DISCUSSION

In this section, the entrainment coefficient and the normalized centerline velocity obtained from LIF-based concentration data are analyzed to investigate the jet behavior and evaluate the proposed indirect methodology. For each experiment, approximately 15–20 data points were selected along the jet centerline, with a spacing between consecutive points ranging from 2 to 5 cm. The corresponding concentration values were extracted from the LIF images and used to estimate the centerline velocity. The entrainment coefficient was then calculated at each downstream position and plotted as a function of the dimensionless distance x/d .

The results indicate that the entrainment coefficient exhibits noticeable fluctuations in the near-field region of the jet, varying approximately between 0.047 and 0.053. This behavior corresponds to the transition region, where the flow is still influenced by the inlet conditions and the self-similar regime is not fully established.

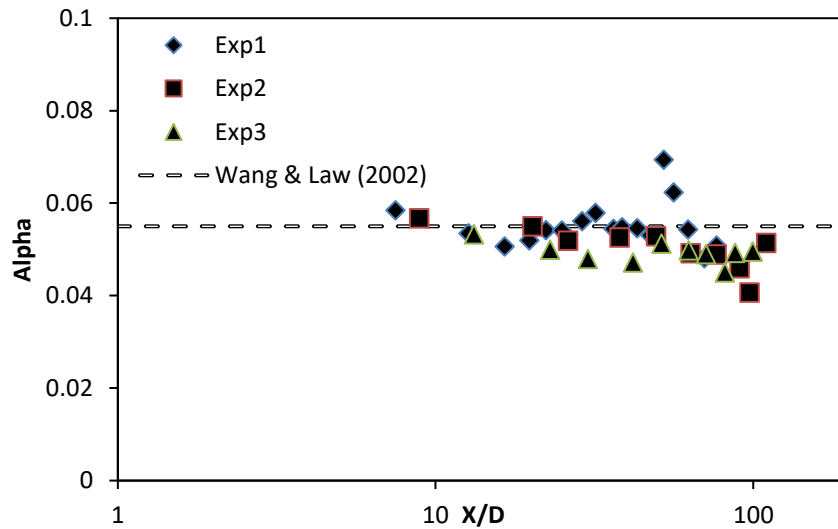


Figure 2- Variation of entrainment coefficient α as a function of the dimensionless distance X/D

As the distance from the nozzle increases, the entrainment coefficient gradually stabilizes and converges to values close to 0.05–0.055. This trend indicates the development of a momentum-dominated self-similar region, where turbulent mixing becomes the dominant mechanism and the influence of the initial conditions becomes progressively weaker. The obtained values show good agreement with classical studies. In particular, both the trend and the asymptotic value of the entrainment coefficient are consistent with Figure 13 reported by Jirka (2004), where a nearly constant entrainment coefficient of approximately 0.05 is observed for axisymmetric turbulent jets in the momentum-dominated regime. This agreement confirms the validity of the present results and supports the reliability of the adopted methodology.

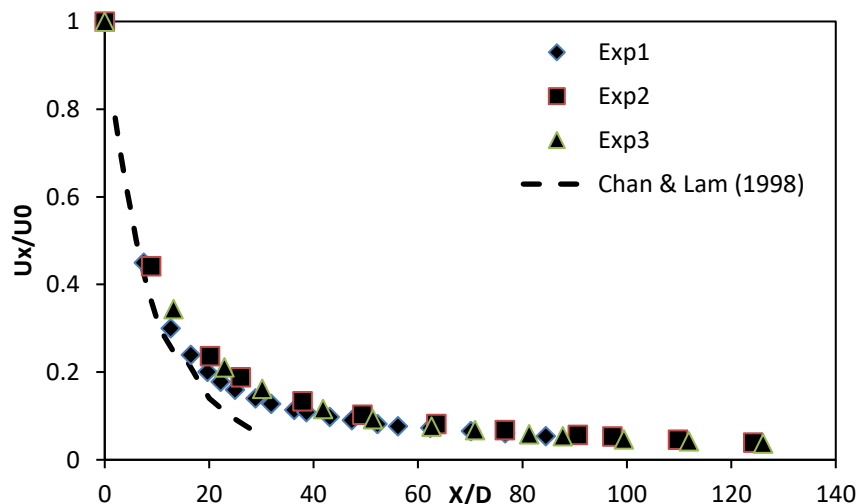


Figure 3-Variation of normalized centerline velocity $\frac{U_x}{U_0}$ as a function of X/D

The variation of the normalized centerline velocity shows a monotonic decrease with increasing distance from the nozzle. This reduction reflects the loss of centerline momentum due to entrainment and mixing with the surrounding fluid, together with the continuous spreading of the jet. The observed decay trend is consistent with the expected behavior of turbulent jets in the self-similar region. The results also show that the evolution of $\frac{U_x}{U_0}$ is nearly identical for all three experimental cases, despite their different Reynolds numbers and inlet velocities. This suggests that, within the analyzed range, the normalized jet behavior becomes less dependent on the initial flow conditions after the development of the self-similar region. However, this interpretation should be limited to the present experimental conditions and to the momentum-dominated region considered in this study.

A combined interpretation of the results shows that the decrease in normalized centerline velocity and the stabilization of the entrainment coefficient occur simultaneously. As the jet spreads and the centerline velocity decreases, ambient fluid is incorporated into the jet, leading to an approximately constant entrainment coefficient in the developed region. Some deviations observed in the near-field region can be attributed to experimental and methodological uncertainties, including image resolution, extraction of concentration values from LIF images, spatial discretization of the centerline points, and the use of numerical differentiation. Despite the limitations, the downstream convergence of α toward values close to 0.055 indicates that the concentration-based approach captures the expected entrainment trend in the self-similar jet region.

CONCLUSION

In this study, a concentration-based approach was applied to estimate the entrainment coefficient in a momentum-dominated axisymmetric turbulent jet. Due to the absence of direct velocity measurements, concentration data obtained from LIF experiments were used to reconstruct the centerline velocity through a proportional relationship between concentration and velocity. Based on this estimated velocity, the entrainment coefficient was calculated using an integral formulation.

The results showed that the entrainment coefficient presents initial fluctuations in the near-field region, followed by a gradual stabilization as the flow develops downstream. In the developed region, the entrainment coefficient converged to values close to 0.055 for the three experimental cases. This behavior is consistent with the classical description of axisymmetric turbulent jets and indicates the development of a self-similar, momentum-dominated region. The normalized centerline velocity also showed a consistent decay with increasing downstream distance, supporting the physical coherence of the concentration-based reconstruction. In addition, the obtained entrainment coefficient presented similar behavior for the different Reynolds numbers, suggesting that the developed-region behavior is less dependent on the initial velocity conditions within the investigated range. However, the proposed approach is based on a simplifying assumption: the proportionality between scalar concentration and velocity. Therefore, the results should be interpreted as an indirect estimate of the entrainment behavior, mainly applicable to the self-similar momentum-dominated region. The lack of direct validation with velocity measurement techniques, such as PIV or LDA, represents an important limitation of the present study.

Overall, the results indicate that LIF-based concentration data can provide a practical indirect approach for estimating entrainment characteristics when direct velocity measurements are not available. Future work should include direct velocity validation and extend the analysis to buoyancy-affected flows and more complex jet configurations.

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