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THEORY OF ENERGY MOTION FOR THE ANALYSIS OF UNSTEADY FLUID FLOW

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ABSTRACT

The theory of energy motion offers a new perspective for analyzing fluid dynamics. The direct problem of the theory involves deriving the laws of fluid motion based on the principles governing energy motion in a fluid. The approach to solving the direct problem is grounded in the fundamental physical principle that a system in stable equilibrium possesses minimal energy. The foundational examples in the implementation of the theory are primarily associated with cases of unsteady fluid motion, although its applicability is not limited to them.

The paper presents information on the principles for implementing the direct problem of the theory, the the principles of energy indication within a fluid, and the forms of energy flow. Within this framework, the structure of long waves in channels becomes significant. The wave is a flow of energy moving in the form of a sequence of planar waves. Some theoretical results obtained for simplified cases of long-wave propagation in infinitely long horizontal channels pertain to the characteristics of such waves, namely, wave velocity and flow velocity. It is noted how wave propagation is influenced by the shape of the channel's cross-section. The unresolved issues highlighted concern the need to establish governing relationships for fluid motion under conditions commonly encountered in practice.

Keywords: unsteady flow; long wave; flow velocity; wave velocity; density of energy; energy flow.

1. Introduction

The aim of this paper is to introduce an application of the theory of energy motion (TEM) in the analysis of fluid flow. This is a classical physical theory (Umov, 1874), which, despite its fundamental nature, has seen limited use, primarily in acoustics. It was within this framework that the principle of energy conservation was first formulated based on a local-cause approach to mechanics, employing notions of energy localization in a medium and its transport in the form of a flow. In the general case, the velocity of energy motion c differs from the velocity υ of the medium itself. Mathematically, the principle of energy conservation is expressed in the form of a continuity equation

$$\frac{\partial e}{\partial t}+\frac{\partial (ec_x)}{\partial x}+\frac{\partial (ec_y)}{\partial y}+\frac{\partial (ec_z)}{\partial z}=0, \tag{1}$$

where e – bulk density of energy [J m^{-3}], t – time [s], x, y, z coordinates [m], c_x, c_y, c_z –components of the energy flow velocity vector [m s^{-1}].

The objective of the theory was formulated as a method to enable the derivation of the equations of motion for the medium itself, based on known general relationships between the distribution and motion of energy within the medium and the displacements of its particles, provided that the laws governing energy motion and distribution had been established experimentally. This approach can be referred to as the direct problem of TEM, which, until recently, had not been explored.

2. Direct problem of TEM for fluids

The possibility of experimentally investigating energy motion in a fluid relies on a fundamental principle of physics stating that a system in a state of stable equilibrium possesses minimal energy, which may be taken as zero. This stable equilibrium is relative and is chosen for the convenience of analysis. A suitable example of such a system is water at rest in a horizontal channel. In this case, the energy density *e* at any cross-section of the channel is zero, and any change in the state of the water or the onset of motion signifies the emergence and localization of energy.

2.1. Localization of energy

Energy can arise only as a result of the work done by external forces. The simplest example is the generation of motion by a vertical paddle (a wave maker). An external force *F* displaces the plate and performs work, transferring momentum and energy to the water. As shown in Fig. 1, the areas of energy localization are the volumes of water on either side of the wave maker, where waves form with corresponding elevations and depressions of the water surface. The change in water level and the onset of motion are indicators of the presence of potential and kinetic energy.

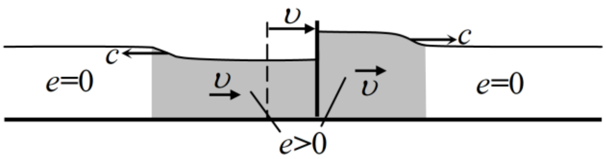

Fig. 1. Localization of energy near a wave-maker.

2.2. Energy transfer

The modes of energy transport manifest in different wave types and are determined by the nature of the wave maker's motion (Fig. 2). According to Russell's experiments (Russell, 1845), energy is localized within the volume of water from the surface to the bottom, confined to the region of surface disturbance, and the energy propagation velocity *c* is equivalent to the wave velocity. Similar conclusions are drawn for long and periodic waves. In all such cases, energy is transported in wave form, characterized by $c \neq \upsilon$. From the perspective of TEM, the primary phenomenon is the motion of energy, while the wave represents the form in which this motion occurs.

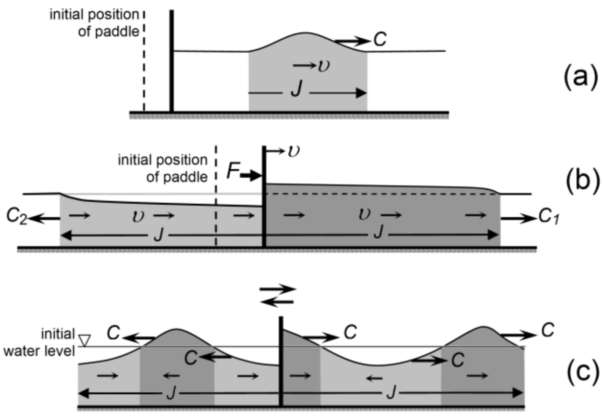

Fig. 2. Energy flow *J* and associated water movement options: a) limited energy portion moves as a solitary wave; b) a continuous energy flow moves as long waves of elevation and depression with wave velocity c_1 and c_2 correspondingly; c) periodical waves.

The long wave model represents it as a continuous sequence of planar waves propagating through volumes of water (Fig. 3). This fundamentally distinguishes it from the Saint-Venant model, which treats a long wave as a superposition of a surface disturbance and a current beneath.

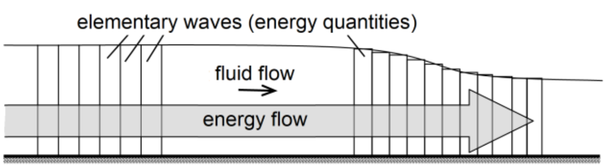

Fig. 3. Energy flow *J* and associated water movement options: a) limited energy portion moves as a solitary wave; b) a continuous energy flow moves as long waves of elevation and depression with wave velocity c_1 and c_2 correspondingly; c) periodical waves.

Each elementary wave carries momentum and energy acquired at the moment of its formation. The energy density is expressed similarly to that of acoustic waves (Landau & Lifshitz, 1987) as $\rho\upsilon^2$, where ρ – density of water, υ - averaged velocity of water flow. The incompressibility condition for each wave yields a simple relationship connecting its geometric and kinematic characteristics.

$$\frac{\upsilon}{c} = 1 - \frac{\omega_0}{\omega}, \tag{2}$$

where ω_0 – area of water body cross-section in equilibrium state [m^2], ω – area of water cross-section of the elementary wave [m^2], c – velocity of the elementary wave equal to the velocity of energy flow [m s^{-1}].

The propagation velocity of an individual elementary wave is related to the density of its energy (Sokolov, 2024)

$$c = \frac{1}{1 - \frac{\omega_0}{\omega}} \sqrt{\frac{2}{\rho} e_U}\ , \tag{3}$$

where e_U - density of potential energy of deformation of water volume in the elementary wave [J m^{-3}]. The momentum and kinetic energy carried by the wave are determined by the average flow velocity υ. Since the kinetic energy constitutes half of the wave energy and equals the potential deformation energy e_U, the average flow velocity is derived from equations (2) and (3) as

$$\upsilon = \sqrt{\frac{2}{\rho} e_U}\ , \tag{4}$$

2.3. Effect of the channel cross-section shape

The shape of the channel cross-section affects the nature of the deformation of water volumes and, consequently, the density of potential energy e_U. Analysis shows that the closer the channel cross-section shape is to a rectangular one, the greater the wave velocity. The flow velocity υ also depends on the shape of the channel cross-section. For example, in a rectangular channel, the velocity is expressed by the equation

$$\upsilon = \eta \sqrt{\frac{g}{h_0 + \eta}}\ , \tag{5}$$

where g – gravity acceleration [m s^{-2}], h_0 – water depth in equilibrium state [m], η - wave height (elevation or falling of water level in relation to initial one) [m]. Equation (5) is an analogue of the Comoy formula (Comoy, 1881), which has a broader range of applications beyond tidal currents.

The structure of a long wave as a sequence of planar waves simplifies calculations, since the flow characteristics are determined by the distribution of energy along the wavelength. This methodology has enabled the formulation of a general approach for evaluating the characteristics of long waves, including in channels with complex cross-sectional shapes (Sokolov, 2020), as well as assessing the impact of channel cross-section shape on discharge capacity (Sokolov, 2022).

3. Unsolved problems

Analytical solutions within the framework of the direct problem of TEM have been obtained for conditions where waves propagate in a horizontal channel of unlimited length. The agreement observed with some known experimental data is encouraging. Nevertheless, these results cannot be considered sufficiently general. Furthermore, the development of the method is limited by the lack of reliable experimental validation.

To achieve results closer to real conditions, the primary focus should be on developing the approach for wave propagation in channels with a slope. A fundamental requirement is the selection of a stable equilibrium state of the fluid in such a channel. From this perspective, flows in natural sloped channels can be represented as depression waves (demand surges). However, specific relationships remain unknown.

Within the energy-based approach, problems involving the propagation of long waves in finite-length channels are of particular interest, where reflection and refraction effects inevitably arise at the boundaries. The nature of the flow becomes more complex as the reflected wave overlaps with the incident wave. From the perspective of TEM, the overall picture is fairly clear: within the same volume of water, there exist counter-directed flows of energy and momentum. Their combination generates a new unsteady flow, which characteristics, wave height and propagation velocity, are unknown. It can only be unequivocally stated that in the case of complete reflection of a long wave from a fixed vertical wall (Fig. 4, a), the flow ceases and the energy density increases. In the case of partial reflection (Fig. 4, b), for example, due to channel narrowing or other conditions, the incident energy flux J_1 divides into partially reflected and transmitted energy fluxes, those are J_2 and J_3 respectively. The superposition of the incident and reflected fluxes creates a region of new flow that propagates upstream, while a refracted new long wave moves downstream. The characteristics of these waves and the resulting water motion in the interaction zone remain unknown.

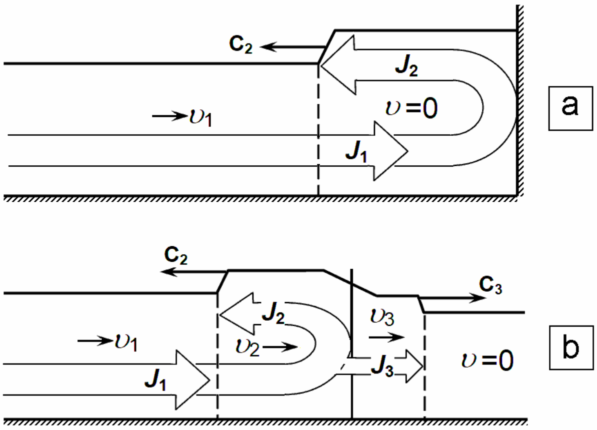

Fig. 4. Effects of long waves in channels of limited length: a) full reflection of the original energy flow; b) reflection and refraction of the energy flow. Indices 1, 2, 3 refer to incident, reflected and refracted wave respectively.

Similar problems related to the interaction of long waves are also of interest. Considering the effects of soliton collisions, where waves pass through each other, analogous phenomena may occur for counter-propagating long waves. During such interactions, waves of elevation or depression may participate in various combinations, with energy fluxes of equal or differing intensities. However, while soliton interactions are brief, the interaction of long waves is expected to occur over a relatively extended period, resulting in the formation of a resultant flow.

Although various types of wave interaction scenarios have been discussed in the literature (Chow, 2009), analyzing these cases from the perspective of TEM may provide new insights.

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