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SIMULATION OF HYDRAULIC TRANSIENTS IN A FICTIOUS RESERVOIR-PIPE-VALVE SYSTEM UTILIZING MOC AND CSPM WITH NORMALIZED DATA

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ABSTRACT

Hydraulic transients are phenomena that occur when the conditions of a flow, such as pressure and velocity, change with time. This regime may not be calculated by explicit solutions. Therefore, it is necessary to use numerical methods, amongst which the method of characteristics (MOC) stands out. The MOC uses a characteristic grid to transform partial differential equations (PDEs) into ordinary differential equations (ODEs). However, as other grid based methods, it may present some problems, especially when there are complex geometries and deformable contours. To solve these problems, there are methods that don't use grids (meshfree), amongst which the corrective smoothed particle method (CSPM) stands out. This method allows that a function and its derivatives may be represented by an approximation. To apply it to a hydraulic transient, it is necessary to make an adaption by considering a term known as artificial viscosity, besides using a temporal integration method. In this paper, in order to verify the efficiency of methods with or without grids, a hydraulic transient was simulated using MOC and CSPM. The results showed that CSPM produces values that are compatible with MOC, although there are little divergences as the simulation runs. Besides, an analysis was conducted to verify the influence of the normalization of velocity and pressure in CSPM.

Keywords: Hydraulic transients; MOC; CSPM; Artificial viscosity; Meshfree methods; Normalization; Shepard´s interpolation.

1. Introduction

Transient flows may not be calculated by explicit solutions. Therefore, it is necessary to use numerical methods. The long-established one is the method of characteristics (MOC), which is grid based, Eulerian and transforms partial differential equations (PDEs) into ordinary differential equations (ODEs). Similarly to other mesh based methods, MOC may present limitations. Most of them are due to the process of establishing the mesh, which should always guarantee that the numerical compatibility condition is the same as the physical compatibility condition for a continuum (Liu and Liu, 2010). In order to cope with these limitations, different meshfree methods were proposed. Their objective is to provide stable and accurate numerical solutions to PDEs with different boundary conditions.

Amongst different meshfree methods, the corrective smoothed particle method (CSPM), developed by Chen et al. (1999), stands out. It is a Lagrangian method based on smoothed particle hydrodynamics (SPH), which was initially conceived to solve astrophysical problems (Lucy, 1977). It defines that a system may be represented by a set of particles containing material properties and with the possibility to interact between one another. This interaction occurs in a range controlled by a smoothing function, also known as Kernel or smoothing Kernel function (Hou et al., 2012).

Between the advantages of CSPM, it is valid to mention its simple computational implementation and applicability to systems with complex geometries and boundary conditions. Nevertheless, the method may present oscillations due to its application to hydraulic transients. In order to prevent them, a dissipation term may be added to the formulation of the method. This term is known as artificial viscosity (Liu and Liu, 2003).

In this paper, in order to compare the results provided by grid based and meshfree methods, MOC and CSPM were utilized to simulate a hydraulic transient in a fictitious reservoir-pipe-valve system. Besides, an analysis was conducted to verify the influence of the normalization of velocity and pressure in CSPM.

2. Mathematical models

2.1. Basic equations

Transient flows are described by the continuity and momentum equations (Chaudhry, 2014):

$$\frac{\partial p}{\partial t}+V\frac{\partial p}{\partial x}+\rho a^2\frac{\partial V}{\partial x}=0 \tag{1}$$

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+\frac{1}{\rho}\frac{\partial p}{\partial x}+gsen\theta+\frac{fV|V|}{2D}=0 \tag{2}$$

where p – pressure [Pa], t – time [s], x – coordinate along the pipe axis [m], ρ – density of the fluid [kg m^{-3}], a – elastic wave speed [m s^{-1}], V – flow velocity [m s^{-1}], g – gravity acceleration [m s^{-2}], θ – pipe slope [rad], f – friction factor, D – inner pipe diameter [m]. Equations (1)-(2) may also be expressed in Lagrangian form:

$$\frac{Dp}{Dt}=-\rho a^2\frac{\partial V}{\partial x} \tag{3}$$

$$\frac{DV}{Dt}=-\frac{1}{\rho}\frac{\partial p}{\partial x}-\frac{fV|V|}{2D} \tag{4}$$

2.2. Method of characteristics (MOC)

To apply MOC, a linear combination of Eqs. (1)-(2) is used. Then, after algebraic manipulations, the positive and negative characteristic equations are obtained:

$$C^+: \frac{dV}{dt}+\frac{g}{a}\frac{dH}{dt}+f\frac{V|V|}{2D}=0 \quad if \quad \frac{dx}{dt}=+a \tag{5}$$

$$C^-: \frac{dV}{dt}-\frac{g}{a}\frac{dH}{dt}+f\frac{V|V|}{2D}=0 \quad if \quad \frac{dx}{dt}=-a \tag{6}$$

where H – piezometric head [m], $dx/dt=\pm a$ – Courant stability condition. Note that Eqs. (5)-(6) are ODEs, whose independent variable is time. Therefore, a time-integration algorithm is applied to a characteristic grid to obtain explicit equations:

$$C^+: H_P=H_A-\frac{a}{gA}(Q_P-Q_A)-\frac{f\Delta x}{2gDA^2}Q_P|Q_A| \tag{7}$$

$$C^-: H_P=H_B+\frac{a}{gA}(Q_P-Q_B)+\frac{f\Delta x}{2gDA^2}Q_P|Q_B| \tag{8}$$

where H_P – piezometric head in point P [m], H_A– piezometric head in point A [m], H_B– piezometric head in point B [m], Q_P – discharge in point P [m³ s^{-1}], Q_A– discharge in in point A [m³ s^{-1}], Q_B– discharge in in point B [m³ s^{-1}], Δx – spatial discretization [m], A – pipe cross-sectional area [m²].

2.3. Corrective smoothed particle method (CSPM)

CSPM works established on two steps (Liu and Liu, 2010): the first one is known as kernel approximation. It consists of representing a function and its derivatives in continuous form as integral representation. The second one is referred to as particle approximation. It works based on the discretization of the computational domain using a set of initial distribution of particles representing the initial settings of the problem. After that, the variables of a particle are obtained by a summation of the values over the nearest neighbor particles.

After applying the two steps, the method provides the particle approximation for a function and its derivatives:

$$f_i = \frac{\sum_{j=1}^{N} f_j W_{ij} m_j/\rho_j}{\sum_{j=1}^{N} W_{ij} m_j/\rho_j} \tag{9}$$

$$f_{i,x} = \frac{\sum_{j=1}^{N} (f_j - f_i) W_{ij,x} m_j/\rho_j}{\sum_{j=1}^{N} (x_j - x_i) W_{ij,x} m_j/\rho_j} \tag{10}$$

where subscripts i and j represent particles i and j, N – number of particles contained in the influence zone of particle x, $f_j = f(x_j)$, m – mass of the particle [kg], ρ – density of the particle [kg m^{-3}], $W_{ij} = W(x_j - x_i, h)$, $W_{ij,x} = (\partial W_{ij}/\partial x_j)$, h – smoothing length [m], $W(x - x', h)$ – smoothing function (Lucy, 1977; Monaghan and Lattanzio, 1985). Observe that the method provides an approximation only to the spatial derivatives of the function, allowing the conversion of a PDE into an ODE. Therefore, it is required to use a time-integration algorithm. One of the most used algorithms is the Euler forward method (Hou et al., 2012), which provides a solution advanced from t^n to t^{n+1} according to:

$$p_i^{n+1} = p_i^n + \Delta t \left(\frac{Dp}{Dt}\right)_i^n \tag{11}$$

$$V_i^{n+1} = V_i^n + \Delta t \left(\frac{DV}{Dt}\right)_i^n \tag{12}$$

The total derivatives are obtained from Eqs. (3)-(4):

$$\left(\frac{Dp}{Dt}\right)_i^n = -\rho a^2 \left(\frac{\partial V}{\partial x}\right)_i^n \tag{13}$$

$$\left(\frac{DV}{Dt}\right)_i^n = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_i^n - \frac{f V_i^n |V_i^n|}{2D} + \left(\frac{\partial(\rho\prod)}{\partial x}\right)_i^n \tag{14}$$

The spatial derivatives are obtained from Eq. (10):

$$\left(\frac{\partial V}{\partial x}\right)_i^n = \frac{\sum_{j=1}^{N} (V_j - V_i) W_{ij,x} m_j/\rho_j}{\sum_{j=1}^{N} (x_j - x_i) W_{ij,x} m_j/\rho_j} \tag{15}$$

$$\left(\frac{\partial p}{\partial x}\right)_i^n = \frac{\sum_{j=1}^{N} (p_j - p_i) W_{ij,x} m_j/\rho_j}{\sum_{j=1}^{N} (x_j - x_i) W_{ij,x} m_j/\rho_j} \tag{16}$$

Note that in Eq. (14) the expression $\frac{\partial(\rho\prod)}{\partial x}$ was added to the momentum equation. It is a dissipation term known as artificial viscosity, whose function is to cope with oscillations from them method. Its formulation is described through three equations (Hou et al., 2012):

$$\left(\frac{\partial(\rho\prod)}{\partial x}\right)_i = \sum_{j \in S_i} m_j \prod_{ij} W_{ij,x} \tag{17}$$

$$\prod_{ij} := \begin{cases} \dfrac{-\alpha a u_{ij} + \beta u_{ij}^2}{\rho}, & (V_i - V_j)(x_i - x_j) < 0 \\ 0, & (V_i - V_j)(x_i - x_j) \geq 0 \end{cases} \tag{18}$$

$$u_{ij} := \frac{\bar{h}_{ij}(V_i - V_j)(x_i - x_j)}{\left|x_i - x_j\right|^2 + \eta \bar{h}_{ij}^2} \tag{19}$$

where α, β and η – constants, $\bar{h}_{ij} = (h_i + h_j)/2$ – constants, $\eta\bar{h}_{ij}^2$ – term whose function is the avoid singularities when i = j. The last step to apply CSPM to a hydraulic transient is to determine the distribution of the set of particles in the system. Usually, it is adopted a uniform distribution along the pipe length.

3. Normalization of velocity and pressure

For the purpose of obtaining results consistent with MOC, the data of velocity and pressure in CSPM were normalized by a process in which a Shepard's Interpolation method (Shepard, 1968) was utilized. It was applied to calculate the derivatives.

4. Analysis of the results

For the hydraulic simulations of the fictious reservoir-pipe-valve system, which was caused by the abrupt closure of the valve, the following parameters were utilized:

Fluid – water, p = 1000 kg/m³, g = 9.81 m/s², L = 20 m – pipe length, Pr = 10^6 Pa – pressure in the reservoir, α = 1.0, β = 2.0, f = 0.02 – constant friction factor, D = 0.797 m, Q = 0.5 m³/s – initial discharge in the pipe, simulation time = 0.3 s, closure of the valve in a single time step, ν = 10^{-6} m²/s – kinematic viscosity of the fluid, h = Δ_x, Δ_x = 0.1 m, number of particles = 201, vertex-centered particle distribution, kernel = cubic spline function. The results are presented in Fig. 1 for the valve section.

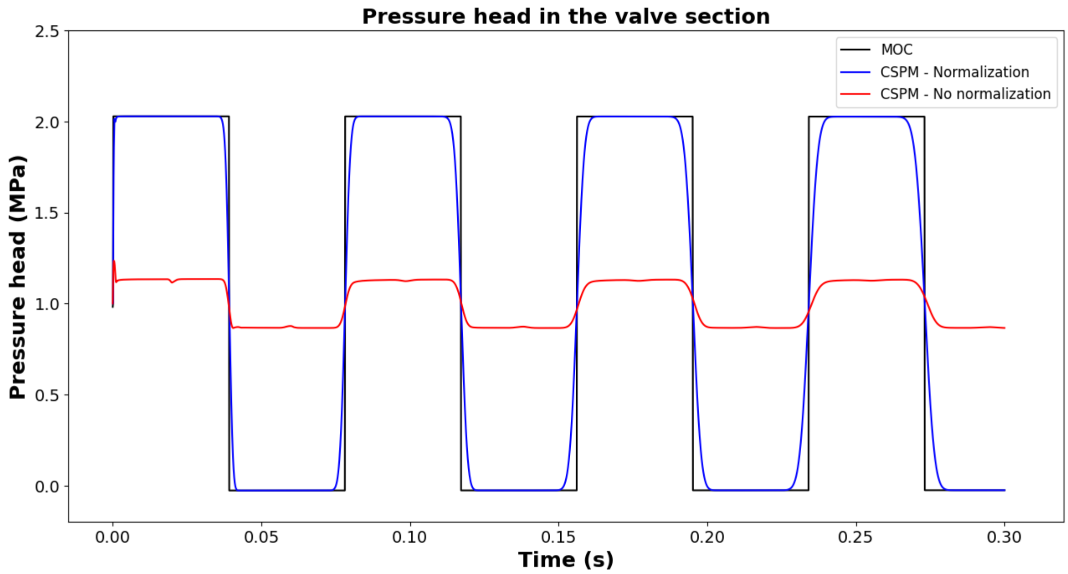

Fig. 1. Pressure head in the valve section for MOC and CSPM

Observe that the results from both methods were similar, especially in the simulation where the normalization was utilized. In the case of no normalization, an overshoot at the first wave is visible. It may be attributed to the numerical error of the CSPM function approximation (Hou et al., 2012). Besides, pressure oscillations were observed in both CSPM simulations. This behavior may be explained by multiple reasons (Pan et al., 2022; Violeau and Rogers, 2016): the adoption of constant pressure wave speed, which does not occur in a real hydraulic transient once the pressure wave tends to decelerate as the simulation proceeds; not considering unsteady friction factor; inaccuracies in the CSPM interpolation procedure itself.

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