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DIFFERENTIATOR-SMOOTHER FILTER FOR TRANSIENT-BASED PIPELINE LEAK DETECTION: A THEORETICAL ANALYSIS

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ABSTRACT

Although transient-based leak detection methods have gained recognition as promising solutions, a consensus on the most effective approach has yet to be reached, underscoring the need for further investigation. This study explores the Differentiator-Smoother (DS) filter, originally developed for fault detection in power transmission lines based on traveling wave theory, as a potential tool for leak detection. A theoretical pressure signal, generated using the Method of Characteristics and modeled on a laboratory-scale hydraulic network, is employed to evaluate the DS filter's ability to extract leak-related information. The results demonstrate that the DS filter can accurately estimate both the location and relative magnitude of leaks. Recommendations for future research are also provided.

Keywords: Differentiator-smoother filter; Leak detection; Hydraulic transients.

1. Introduction

Leak detection in water distribution networks has become a central concern for water utilities, given its direct impact on operational efficiency, sustainable water resource management, public health, and environmental protection. Leaks, in addition to posing risks to consumer health due to possible water contamination, often indicate maintenance deficiencies and lead to considerable economic losses for the utility, mainly because of lower revenue from billed water, increased energy consumption for pumping, and the waste of chemicals used in water treatment. Furthermore, in systems that carry hazardous fluids, leaks can cause serious environmental damage and compromise nearby infrastructure.

In this context, several traditional leak detection methods are commonly used, such as visual inspection, acoustic techniques and water balance calculations. However, these approaches have notable drawbacks, such as slow response time, high operational costs and limited use of modern technologies like in-field monitoring devices and hydraulic modeling (Wang et al., 2001; Romero-Ben et al., 2023). Efficient leak detection has therefore become a key aspect in developing more advanced and cost-effective solutions (Farah and Shahrour, 2024). Given the limitations of conventional techniques, transient-based leak detection methods have gained increasing attention.

A leak causes disturbances in the pressure wave reflections generated by a hydraulic transient, making it possible to detect through event analysis (Ayati et al., 2019). These methods are generally divided into two main categories: signal-processing-based methods and hydraulic-modelling-based methods. The former is easier to apply but more suited to simple systems and configurations, while the latter requires modeling efforts that match the system complexity (Che et al., 2021). Despite many existing approaches, there is still no consensus on the most effective method, leaving room for new solutions. This study presents a preliminary investigation of a new signal-processing-based technique — the Differentiator-Smoother (DS) filter — which was originally developed for fault detection in electric power transmission lines. The method is tested on a pressure signal generated by a theoretical hydraulic transient in a reservoir–pipe–valve (RPV) system with a single leak. The objective is to assess the filter capacity to identify and extract key features related to the leakage, such as its location and magnitude.

2. Leak detection based on transient wave reflection

Assuming a constant pressure wave velocity, the leak location can be determined using Eq. (1), which applies to an RPV system with a pressure sensor at the valve and a position reference at the upstream end:

$$x_L = L - a\frac{\Delta t_L}{2} \tag{1}$$

where x_L – leak location [m], L – pipe length [m], a – pressure wave speed [m s^{-1}], and Δt_L – time between the transient initiation and the arrival of the reflected wave [s].

To quantify the leak, Vítkovský (2001) proposed a formulation for calculating the leakage coefficient $C_d A_L$ based solely on pressure measurements taken during the transient event, as shown in Eq. (2):

$$C_d A_L = \frac{A}{a}\sqrt{\frac{g}{2}}(H_T - H_L)\left(\sqrt{\frac{1}{2}(H_T + H_L)} - \sqrt{H_0}\right)^{-1} \tag{2}$$

where $C_d A_L$ – leakage coefficient [m^2], A – cross-sectional area of the pipe [m^2], g – gravitational acceleration [m s^{-2}], H_T – peak pressure head from the first wave [m], H_L – pressure head after the leak-induced drop [m], and H_0 – steady-state pressure head [m]. Using the orifice equation $Q_L = C_d A_L\sqrt{2gH_0}$ and dividing by Q_0, where Q_L – leak flow rate [m^3 s^{-1}] and Q_0 – steady-state flow rate [m^3 s^{-1}], the relative leak flow can be obtained by Eq. (3), expressed as a percentage of the initial flow:

$$\frac{Q_L}{Q_0} = \frac{gA}{a}(H_T - H_L)\sqrt{H_0}\left(\sqrt{\frac{1}{2}(H_T + H_L)} - \sqrt{H_0}\right)^{-1} \cdot 100\% \tag{3}$$

3. Differentiator-Smoother filter

Originally proposed by Nathanson et al. (1969) and later applied to fault detection in power transmission lines by Ando et al. (1985), the DS filter is used to detect singularities through traveling wave analysis. It operates in a two-step procedure: smoothing and differentiation. The first step reduces high-frequency components that could mask a reflected wave, while the second transforms the singularity into a pulse-shaped output. Two properties of the DS filter are essential to highlight: (i) the output pulse occurs consistently regardless of the amplitude of the step variation generated by the reflected wave; and (ii) the filter output preserves the polarity of the variation — that is, if the variation is positive (or negative), the output is also positive (or negative) (Schweitzer et al., 2014).

The DS filter is a discrete, windowed filter shaped like a step function. It is defined by two parameters: N_{DS} and G. Based on these parameters, the filter consists of $(N_{DS} - 1)/2$ coefficients of value $-G$, a central coefficient of zero value and $(N_{DS} - 1)/2$ gain coefficients of value $+G$. The window duration T_{DS} determines the number of coefficients N_{DS}, calculated as $N_{DS} = \lfloor T_{DS}/\Delta t \rfloor$. Lopes et al. (2019) provide further details on the DS filter output. If the input variation resembles a step function, the filter produces a triangular output. As the step becomes flatter, the output transitions into a parabolic shape with a reduced peak. Additionally, when the output is triangular, choosing $G = 2/N_{DS}$ ensures that the triangle height matches the amplitude of the input variation.

4. Method

As previously mentioned, the aim of this study is to evaluate the use of the DS filter for leak detection under transient flow conditions. The practical application of the DS filter to the first pressure wave is illustrated in Fig. 1, where the filter positive pulse output has coordinates (t_p^+, H_p^+) and the negative pulse has coordinates (t_p^-, H_p^-), where H_p^+ – pressure head corresponding to the DS filter positive pulse output [m] and H_p^- – pressure head corresponding to the DS filter negative pulse output [m]. Let t_V – time at which the valve closes [s], t_L – time at which the reflected wave is detected [s], t_p^+ – time corresponding to the DS filter positive pulse output [s] and t_p^- – time corresponding to the DS filter negative pulse output [s]. Thus, the time interval Δt_L can be expressed as $\Delta t_L = t_V - t_L = t_p^+ - t_p^-$.

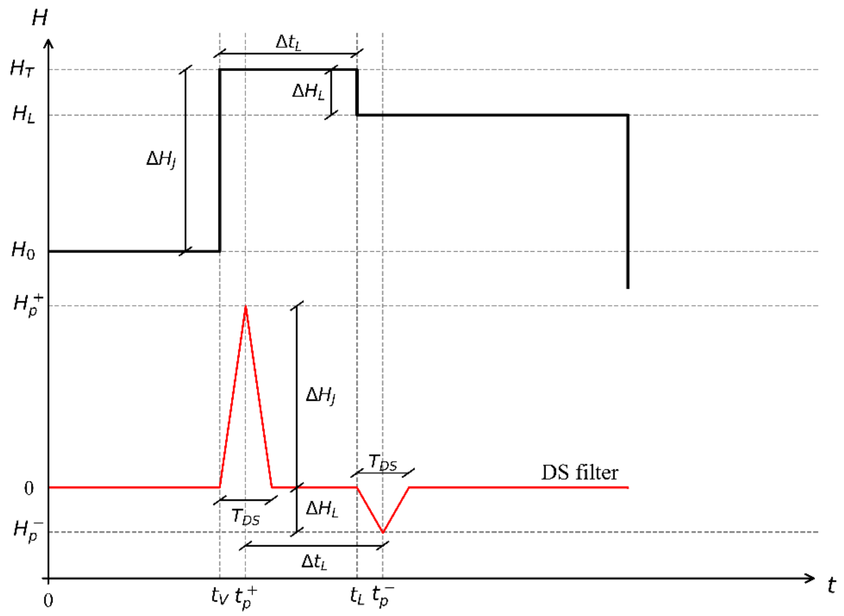

Fig. 1. Schematics of the DS filter output for transient pressure signal

Considering the Joukowsky overpressure $\Delta H_J = H_T - H_0$, the pressure drop due to the leak $\Delta H_L = H_T - H_L$ and applying the relationships $H_p^+ = \Delta H_J$ and $H_p^- = -\Delta H_L$, Eq. (3) can be rewritten as Eq. (4). This formulation enables the calculation of the relative leak flow rate using pipe characteristics, the steady-state pressure head, and the DS filter output:

$$\frac{Q_L}{Q_0} = \frac{gA}{a}\left(-H_p^-\right)\sqrt{H_0}\left(\sqrt{H_0 + H_p^+ + \frac{1}{2}H_p^-} - \sqrt{H_0}\right)^{-1} \cdot 100\% \quad (4)$$

A theoretical pressure signal was generated numerically using the Method of Characteristics (MOC), based on the copper laboratory hydraulic network at Instituto Superior Técnico (Lisbon, PT), which follows the RPV model. Some of the system most relevant parameters are: L = 15 m, D = 0.02 m, where D – internal pipe diameter, a = 1255 m s^{-1}, H_0 = 45.6 m and Q_0 = 109·10^{-6} m^3 s^{-1}. The simulation was carried neglecting head losses and assuming an instantaneous valve closure. One leak was simulated with magnitude Q_L/Q_0 = 20% and position x_L = 12 m. Further details about the network are provided in Soares et al. (2015). A time step of Δt = 10^{-6} s was used for the MOC simulation, and a window duration of T_{DS} = 3·10^{-3} s was adopted for the DS filter application.

5. Results

Figure 2 illustrates the application of the DS filter to the theoretical pressure signal.

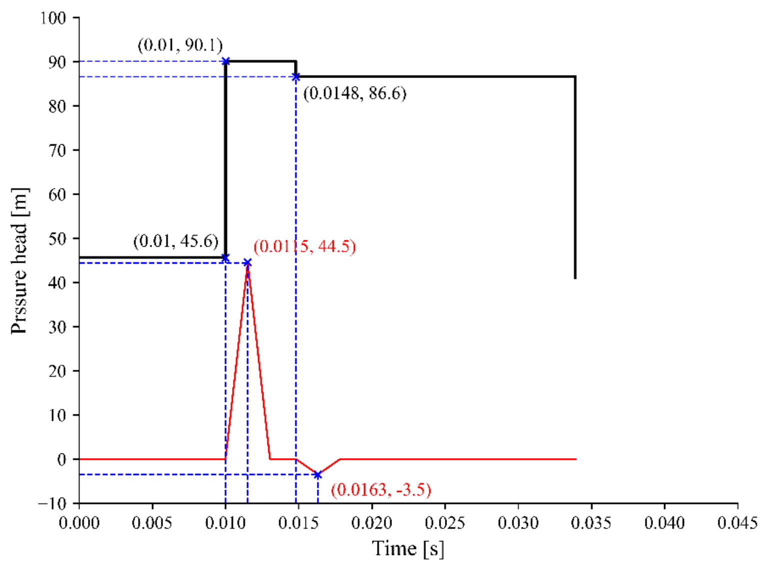

Fig. 2. DS filter output for theoretical transient pressure signal

Hence, the filter outputs are displayed in Table 1.

Table 1. DS filter outputs

H [m]		t [s]	
H_p^+	44.5	t_p^+	0.0115
H_p^-	-3.5	t_p^-	0.0163

Accordingly, using Eq. (1), and knowing the pipe length L, the pressure wave speed a and the time interval Δt_L corresponding to the detection of the reflected wave — where the latter is obtained from the DS filter output — the leak position is calculated as x_L = 11.99 m. Similarly, by knowing the parameters g, A, a, H_0, and Q_0, and determining H_p^++ and H_p^- from the DS filter output, the relative leak flow rate can be calculated using Eq. (4), resulting in Q_L/Q_0 = 20,1%.

In addition, it can be observed that the Joukowsky overpressure ΔH_J and the pressure drop due to the wave reflected at the leak point ΔH_L — which are the two main variations in the pressure profile — are represented with equal magnitude and polarity by the DS filter pulses. Likewise, it can be noted that the time between valve closure and the arrival of the reflected wave, and their respective peaks in the DS filter output, satisfies the relation $t_V - t_L = t_p^+ - t_p^-$.

6. Conclusion

This study aimed to perform an initial analysis of the application of the Differentiator-Smoother filter for leak detection in hydraulic networks under transient conditions. For this purpose, based on data from a laboratory pipe rig located at Instituto Superior Técnico (Lisbon, PT), a theoretical pressure signal was generated using the Method of Characteristics, neglecting head losses and assuming instantaneous downstream valve closure. With a hydraulic simulation time step of Δt = 10^{-6} s, the Differentiator-Smoother filter was applied using a window duration of T_{DS} = $3 \cdot 10^{-3}$ s, from which the two filter parameters — the number of coefficients N_{DS} and the gain G — were obtained.

The filter output proved consistent with theoretical expectations, producing pulses with the same magnitude and polarity as the variations in the pressure signal. Moreover, it enabled precise identification of the time instants corresponding to pulse detection. As a result, it was possible to determine both the leak location and its flow rate relative to the steady-state flow, with both outcomes matching the expected values. Finally, the Differentiator-Smoother filter demonstrated promise as a tool for leak detection in pipeline systems modeled under the RPV approach. For future work, it is suggested that the filter be applied to real leakage scenarios and other system configurations to evaluate its performance beyond theoretical conditions.

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